Artefacts, and in particular ICT tools, are commonly considered as resources susceptible of enhancing the teaching-learning activity. But while the potentialities of these artefacts for learning have been extensively studied with a main focus on the their possible use by the students and the consequent benefits for them, there has been the tendency to underestimate the complexity of the teacher’s role in exploiting these potentialities.

In this chapter we address the issue how an artefact, and more specifically an ICT tool, can be a resource for the teacher, that is how it might support the accomplishment of the teacher’s educational tasks.

We will start presenting a specific theoretical framework aimed at describing and modelling the teaching-learning process based on the use of a specific artefact. Such theoretical approach explicitly takes into account the role of the teacher, thus it provides the base for an explicit model of what is expected from him.

The model elaborated in (Bartolini Bussi and Mariotti, 2008) is centred around the seminal idea of semiotic mediation introduced by Vygotsky (1978) and aims to describe and explain the process that starts with the student’s use of an artefact and leads to the student’s appropriation of a particular mathematical content.

In this chapter we intend to refine this model with the aim of highlighting the specific role played by the artefact as resource for the teacher in accomplishing his educational task. In this perspective we focus on how the teacher can use the artefact according to his specific educational goals. Taking an instrumental approach (Rabardel, 1995) and exploiting the notion of utilization schemes we describe how an artefact may function as an instrument of semiotic mediation for a teacher. More specifically, the focus of this chapter is on the teacher’s actions mediated by the artefact and concerning the orchestration of the classroom discussion.

5.1 Mediation and teaching-learning according to a Semiotic Approach

The term “mediation” is frequently used with different and not always compatible meanings. But it is in relation to the use of artefacts, and specifically to the introduction of new technologies in school practice, that the term has become widely present in the current mathematics education literature (Meira, 1995; Radford, 2003; Noss and Hoyles, 1996; Borba and Villarreal, 2005). The idea of mediation is employed to refer to the potentiality of a specific artefact in respect of fostering learning processes. The participation of an expert to the mediation process is rarely considered, mostly the mediation potential refers to the accomplishment of a task through the use of the artefact. However, what is often left implicit is the epistemological issue concerning the relationship between accomplishment of a task mediated by the artefact and learning. Many teachers, and researchers as well, apparently believe that the mathematical meaning evoked by the use of the artefact is sufficiently transparent to students and that therefore the situation does not warrant intensive mediation.

The Theory of Semiotic Mediation (TSM) elaborated in (Bartolini Bussi and Mariotti, 2008) addresses this issue. It combines a semiotic and an educational perspective, and elaborates on the notion of mediation considering the crucial role of human mediation (Kozulin, 2003, p.19) in the teaching-learning process.

Taking a semiotic perspective means to interpret teaching-learning recognizing the central role of signs, both as a product and as a medium, in the construction of knowledge. Consequently, specific attention is to

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1 To appear in the french version in

2 The use of the term “sign” is inspired by Pierce. We assume an indissoluble relationship between signified and signifier. In the stream of other researchers (Radford, 2003; Arzarello, 2006) we developed the idea of meaning originate in the intricate interplay of signs (Bartolini Bussi and Mariotti, 2008).
be paid to the processes of production of signs and of their transformation, which in turn is considered as an evidence of learning. How fostering or guiding this process is a crucial issue. In the following sections we outline how one can organize a teaching-learning sequence integrating the use of artefacts, based on the notion of semiotic mediation. Such description is developed around the key notions of **semiotic potential of an artefact** and of **didactic cycle**. Within this frame we describe different mediation modalities that may be accomplished by the teacher in exploiting the semiotic potential of a given artefact.

### 5.2 The semiotic potential of an artefact

Following Hoyles (1993), one can speak about the relationship between artefact and knowledge as evoked knowledge. For experts the artefact may evoke a specific knowledge corresponding to what is mobilized to solve specific problems. For example, the positional notation of numbers may be evoked by an abacus (Fig. 1); similarly, a Dynamic Geometry System may evoke the classic "rule and compass" geometry. But certainly there is the need of distinguishing between meanings emerging from the practice based on the artefact use and mathematics knowledge developed in relation to such use. With this respect the case of the abacus is paradigmatic: centuries of practice of computation with the abacus were not sufficient to trigger the move towards the positional notation system for numbers.

The notion of **semiotic potential of an artefact** is meant to capture and make explicit that distinction.

By **semiotic potential of an artefact** we mean the double semiotic link which may occur between an artefact and on the one hand the personal meanings emerging from its use to accomplish a task, and on the other hand the mathematical meanings evoked by its use and recognizable as mathematics by an expert.

The distinction between personal meanings and mathematical meanings may remind of Brousseau’s distinction between knowing (connaissance) and knowledge (savoir) (1997). Even if they are not in antithesis the two perspectives cannot be reduced one to the other: the former stresses the semiotic dimension of the teaching-learning processes which is in the shadow in the latter.

In a mathematics class context, when using an artefact for accomplishing a task, students can be led to produce signs which can be put in relationship with mathematical signs. But the construction of such relationship is not a spontaneous process. On the contrary it should be assumed as an explicit educational aim by the teacher, who can orient his own action towards the promotion of the evolution of signs expressing the relationship between artefact and tasks into signs expressing the relationship between artefact and knowledge (Fig. 2).

![Fig. 2. Semiotic potential of an artifact and teacher’s mediation](image)

This chapter focuses exactly on the teacher’s actions intentionally oriented towards the promotion of the evolution of signs in the orchestration of the classroom discussions (§ 5.4).
As Sensevy points out (chapter 8 of this volume) investigating the teacher’s didactical intentions has a crucial importance for understanding his didactical actions. In order to describe intentions Sensevy proposes a frame articulated in several dimensions. We mention two of them which are especially relevant to our study: the existence of a link between intentions and resources, and the possibility of describing intentions at different levels of granularity, in particular the distinction between préalable intentions and intentions en action. The former is reflected in the fact that the analysis of the semiotic potential of the artefact contributes to the definition of the teacher’s didactical intentions. As for the latter, we will see in § 5.7 that the teacher’s action cannot be designed completely a-priori. The orchestration of the classroom discussion demands a number of on the spot decisions reflecting the didactical intentions en action which in turn result from the relationship between the préalable didactical intentions and the actual development of the discussion.

The semiotic potential of an artefact can be identified through an analysis involving at the same time a cognitive and epistemological perspective. The richer the description of the semiotic potential is, the richer is the base for the design of teaching-learning sequences centred on the use of a given artefact. This issue can be inscribed in the wider problématique of the relationship between the resources available to teacher and the didactical decisions he takes in the design phase. In the chapter 15 of this volume, within the frame of the French research paradigm Trgalova investigates the processus de prise de décisions didactiques par le professeur en situation de projet de séance en relation avec les ressources to which she/he can access.

In the case of ICT, the instrumental approach (Rabardel, 1995; Trouche, 2005; Artigue, 2002; Lagrange, 2000) allows one to articulate the use of the artefact with the different tasks and provides the base for a cognitive and epistemological analysis that contributes to identify the potential meanings, which might emerge during students’ activities, and the related mathematical meanings. Using the terminology introduced by Winslow (2003, p. 275), the notion of semiotic potential stresses the role that an artefact may have as a “vehicle for learning”, and explicitly addresses the relationship between pragmatic and didactic issues.

5.3 The didactic cycle

How does it happen that a person might become conscious of the meanings emerging from the artefact use and might relate them to mathematics? In other words, using a terminology inspired by Leont’ev (1964/1976), how may happen that personal meanings arising in the accomplishment of a task mediated by an artefact may become mathematical meanings?

According to the TSM, the teaching-learning process starts with the emergence of students’ personal meanings in relation to the use of the artefact. The emergence is witnessed by the appearance of specific signs. They are situated signs, strictly linked to the use of the artefact (artefact signs). Once personal signs emerged, their evolution into the mathematical signs has to be fostered by the teacher through specific social activities. In summary, the process of semiotic mediation consists in the evolution process that has its first step in the emergence of personal meanings related to the accomplishment of a task and develops in the collective construction of shared signs related to both the use of the artefact and the mathematics to be learnt. Such evolution can be promoted through the iteration of didactic cycles where different categories of activities take place, each of them contributing differently but complementarily to develop the complex process of semiotic mediation.

- **Activities with the artefact.** This type of activity constitutes the start of a cycle and is based on tasks to be carried out with the use of an artefact. Situations are designed with the aim of promoting the emergence of signs referred to artefact use.

- **Activities of individual writing.** Students are involved individually in different semiotic activities concerning written productions. For instance, students might be asked as homework to write individual reports on the previous activity with the artefact, reflecting on their own experience, and raising possible doubts or questions. Written productions can become objects of discussion in the following collective work.

- **Classroom Discussion.** Discussions play an essential part in the teaching-learning process and constitute the core of the semiotic process, on which teaching-learning is based. The whole class may be engaged: for instance, after problem solving sessions the various solutions may be discussed collectively; but also students’ written texts or other texts may be analysed and commented. The main objective of teacher’s action in such a discussion is that of fostering the move towards mathematical meanings, taking into account individual contributions and exploiting the semiotic potentialities coming from the artefact use.

In other words, assuming a semiotic mediation perspective calls for the establishment in the teaching practice of a specific activity format (Burns and Anderson, 1987; quoted in Ruthven, chapter 10 of this volume) consisting exactly of the iteration of didactical cycles. Ruthven shows that the integration of ICT tools into everyday teaching entails important modifications of well-established teaching practices. More
5.4 The role of teacher

The previous discussion intended to highlight the essential role played by the teacher whose action is crucial at every step of the didactic cycle. When the teacher succeeds in exploiting the potentialities discussed so far, one says that the artefact is used by the teacher as a tool of semiotic mediation (Bartolini Bussi and Mariotti, 2008, p.754). In other words, the semiotic mediation perspective stresses the fact that the artefact is used in the classroom not only by the students for solving given (mathematical) tasks, but also by the teacher for accomplishing a didactical task: pursuing his educational goals. In this sense, the artefact is a resource for the teacher.

As anticipated, the focus is on the teacher's actions mediated by the artefact and concerning the orchestration of the classroom discussion. The metaphor of orchestration is commonly used when addressing the issue of integrating tools in the classroom. But the metaphor is not used to convey the same ideas by the different authors. In particular, we use the expression orchestration of the classroom discussion to refer to the teacher's action to manage the classroom discourse after Bartolini Bussi (1998) who introduces the metaphor of orchestration together with that of a classroom discussion as “a polyphony of articulated voices on a mathematical object that is one of the motives for the teaching-learning activity” (op.cit., p. 68). Hence, the term “orchestration” here has a meaning different (though not antithetical) from that of instrumental orchestration defined as the intentional systematic organization of both artefacts and humans of a learning environment for guiding the instrumental geneeses for students (Trouche 2005, p. 126). On the one hand, the orchestration of the classroom discussion can be related to what Drijvers labels the didactical performance component of the instrumental orchestration (Drijvers et al. 2009). On the other hand, the objective of the orchestration of the classroom discussion is not that of guiding students’ instrumental genesis, but that of developing shared meanings having an explicit formulation, de-contextualized from the artefact use, recognizable and acceptable by the mathematicians’ community.

This specification is needed to avoid ambiguities; in fact in the next sections we are going to recall some ideas coming from the instrumental approach which will be used as a lens to look at the teacher's use of the artefact in the orchestration of the classroom discussion as framed within the TSM. More precisely, we are going to re-formulate what we mean by the expression “the teacher uses the artefact as a tool of semiotic mediation”, according to an instrumental approach.

5.5 The notion of scheme and the instrumental approach

The distinction between an artefact and its use for solving a task is at the core of the instrumental approach, and it is reflected in the distinction between artefact and instrument. The former is the material or symbolic object in se designed for specific purposes, while the latter refers to the mixed entity “made up of both artefact-type components and schematic components that we call utilization schemes. This mixed entity is born of both the subject and the object.” (Rabardel and Samurçay, 2001).

The notion of utilization schemes comes after the re-elaboration of the piagetian original notion of scheme made by Vergnaud, who characterizes it as an invariant organisation of the activity for a given class of situations\(^3\). More precisely, Vergnaud (1990, p.159) defines a scheme as:

«une totalité organisée, qui permet de générer une classe de conduites différentes en fonction des caractéristiques particulières de chacune des situations de la classe à laquelle il s’adresse. Cela n’est possible que parce que le schéme comporte:

- des invariants opératoires (concepts-en-acte et théorèmes-en-acte) qui pilotent la reconnaissance par le sujet des éléments pertinents de la situation, et la prise d’information sur la situation à traiter;
- des anticipations du but à atteindre, des effets à atteindre et des étapes intermédiaires éventuelles;
- des règles d’action de type si… alors… qui permettent de générer la suite des actions du sujet;

\(^3\) In this chapter we use the term situation after Vergnaud (1990, p.151): «toute situation peut être ramenée à une combinaison de relation de base avec des données connues et des inconnues, lesquelles correspondent à autant de questions possibles.». In Vergnaud (op.cit.) the reader can find a short discussion on the differences between this sense of the term “situation” and the sense given to it by Brousseau (1997).
• des inférences (ou raisonnement) qui permettent de calculer les règles et les anticipations à partir des informations et du système d'invariants opératoires dont dispose le sujet.

Even if all the components of a scheme are important, operational invariants have a prominent role. They consist of the implicit knowledge which structures the whole scheme: they drive the identification of the situation and of its relevant aspects, and allow selecting suitable goals and inferring the rules for generating the appropriate actions to achieve those goals.

Now, if we consider the teacher’s use of the artefact as framed in the TSM, and we look at it through an instrumental perspective, we can re-formulate the expression “the teacher uses the artefact as a tool of semiotic mediation”, by saying that the artefact functions an instrument for the teacher to accomplish a mediation task; that is the teacher uses the artefact according to specific utilization schemes. We synthesize the two points of view through the expression instrument of semiotic mediation.

5.5.1 Inferring utilization schemes from observation

We are now going to elaborate on the teacher’s use of an artefact in the classroom, i.e. to describe specific utilization schemes which, according to the TSM, the teacher is expected to develop for constructing an instrument of semiotic mediation.

Through a number of teaching experiments we collected several examples of teachers’ effective use of the artefact, that is examples of artefact use which according to our analyses promoted effectively the semiotic mediation process. Starting from those examples we attempted a description of the possible utilization schemes constructed by the teachers; that is an a-posteriori characterization of possible utilization schemes which according to the TSM might be developed for constructing an instrument of semiotic mediation.

This procedure raises the crucial methodological issue of how schemes and in particular utilization schemes of an artefact can be inferred from observation.

As for inferring schemes tout-court, the leading idea is to look for regularities in the individual’s behaviours across a number of situations: Bourmaud (2006, p.41), after Zanarelli (2003), holds the need of investigating the following dimensions of the activity: « […]
• des régularités de séquencés d’activité;
• l’existence d’un choix entre plusieurs possibilités ;
• la transformation de la situation, à savoir les effets de l’activité sur la situation;
• l’opérationnalité, c’est-à-dire la performance de l’activité.»

When an artefact is present, the analysis needs to take into account more elements besides those mentioned above. It is still Bourmaud (op.cit. p.41) who highlights the need of considering also the following aspects: « […]
• des artefacts utilisés par les opérateurs;
• de classe de situations dans lesquelles sont utilisés ces artefacts;
• des organisations invariantes de l’activité, pour en inférer les schèmes sous-jacents;
• des fonctions attribuées aux artefacts par les observateurs, par une approche fonctionnelle;
• de l’objet sur lequel l’artefact permet d’effectuer des modifications.»

In our analysis we will try to make explicit these dimensions as far as possible.

5.6 The teaching experiment

The description of the teacher’s utilization schemes (§ 5.7) will be illustrated through examples drawn from a two months long - teaching experiment centred on the use on Cabri, which involved Italian and French 10th grade classes, and was repeated for three school years (for a detailed description of the sequence see Falcade, Laborde and Mariotti 2007; Falcade 2006).

The educational goal was that of using Cabri (Laborde and Bellemain, 1995) as a tool of semiotic mediation for introducing students to the idea of function as co-variation. The design of the sequence of activities was consistent with the structure of the didactic cycle. Students’ productions and audio-recordings of classroom activities were collected and analysed. In the following, after a short description of the semiotic potential, on
which the educational intervention was designed, we give a specific account of the first phase of the sequence from which the illustrative examples are drawn.

It is possible to identify a rich system of meanings emerging from the use of a Dynamic Geometry Environment (DGE), such as Cabri, and the corresponding system of meanings related to the mathematical notion of function. On the one hand we consider certain components of Cabri and their use, such as: basic points and points obtained through a construction, the dragging tool and its effect on the different kinds of points, the trace tool and the effect of its activation, the macro tool and its functioning with respect to a construction. On the other hand we consider the mathematical notion of function and all the related notions such as that of independent and dependent variables, parameter, domain, image, and finally that of graph. We will give a brief account of this related system.

Motion obtained through the use of the dragging tool constitutes the main feature of a DGE. Points can move in two main ways: basic points can move after a direct action by means of the dragging tool; whereas constructed points move as a consequence of dragging the basic points of the construction. In this way the use of the dragging tool allows one to experience the combination of two interrelated motions: the motion of constructed points depending on the motion of the basic ones through a relation stated by the construction. As a consequence, the use of the dragging tool may be considered in relation to the idea of function as co-variation between dependent and independent variables.

Other features of Cabri can be identified offering a semiotic potential with respect to the notion of function: for instance, the macro tool that realizes a given construction. Whenever applied to the required “initial elements” the macro will produce the corresponding “final elements”. That clearly evokes the idea of function as the relation between input and output.

As said, we concentrate on the first collective discussion of the sequence. The discussion comes after the first activity with Cabri devoted to explore the effect of a macro-construction. Pupils working in pairs were asked to apply to three given points (A, B and P) an unknown macro-construction generating a fourth point (H). Then they had to explore systematically the effect of dragging a point and produce a common written description of the movement of the different points. The aim of the discussion is to introduce the notion of variation and co-variation and to produce a shared formulation of a definition of function as co-variation in geometrical settings.

5.7 Utilization schemes of an artefact for the orchestration of a mathematical discussion

Now we can describe some possible utilization schemes which the teacher should develop for constructing an instrument of semiotic mediation. These schemes are identified from the observation of several examples of teacher’s effective uses of the artefact. The examples given below come from the teaching experiment described in the previous section, but the discussion is meant to have a more general scope.

We consider different classes of situations related to the activity family of collective discussions within the didactical cycle. Following Bourmaud (2006, p.40), by activity family we mean an «ensemble des classes de situations qui correspondent à un même type de finalité générale de l’action». The general objective of the classroom discussion within a didactical cycle is for a teacher to promote both the emergence of the students’ personal signs and their evolution towards the desired mathematical signs. Because this evolution takes place, the active engagement of students is needed: it is not enough for the teacher to make explicit the mathematical knowledge which students should learn and to link it to the solution of the given tasks. The crucial point is to develop a semiotic link between shared meaning emerging from personal experience and mathematical meanings that for their nature are not negotiable but culturally established. The semiotic process that is going to occur during a collective mathematical discussion constitutes the core of the process of semiotic mediation assumed by the TSM; it involves the teacher and the students, the artefact use and the mathematics.

Two distinct classes of objectives can be recognized in a classroom discussion: the joint construction of shared signs and the evolution of these signs towards mathematical signs. The analysis of the teacher’s utilization schemes of the artefact is going to be articulated according to these two objectives.

The description of the schemes will be given in terms of class of situations, set of objectives, and system of operational invariants, while we will leave out the description of the rules of action and inferences. In fact we think that the description of operational invariants (together with that of situations and objectives) allows capturing the essence of the scheme: «c’est le concept d’invariant opératoire qui permet de faire le lien entre la form opératoire et la forme predicative de la connaissance, justement parce qu’il s’agit de la composante épistémique du schème, celle qui soutient en dernier ressort l’organisation de l’activité» (Vergnaud, 2005, p.129). On the other hand, as Lagrange (1999, p.58) underlines, “being adaptive mental constructs, schemes cannot be entirely described in a rational form”. And anyway one can describe the different components integrated into a scheme only through the categories of the explicit knowledge (Vergnaud,
1990, p.145). Hence, there is the risk that the explicit formulation of all the components of a scheme results in repetitions and redundancies, without doing justice to their different cognitive functionalities, and while overloading the whole discussion.

5.7.1 Joint construction of shared-signs

Consider the objective of promoting the emergence of personal signs referring to the common experience with the artefact. Two utilizations schemes can be related to this objective: the back to the task scheme and the focalization scheme.

The “back to the task” scheme

During a classroom discussion a crucial situation occurs when the production of signs by the students should start and then be promoted by the intervention of the teacher. Hence, a first class of situations can be characterized as follows:

- By S1 we define the class of situations characterized by the need of promoting the students’ production of signs related to the actual use of the artefact for accomplishing a given task.

In general, the very beginning of a classroom discussion belongs to S1, as well as the impasses that can occur during the discussion: for instance, when the students’ contributions begin to fail or cease to adequately feed the classroom discourse. In short, in all those moments in which the production of signs should start or re-start.

These situations demand of an explicit intervention of the teacher, whose actions, keeping in mind the global objective of the classroom discussion (that is the préalable didactical intention), can be aimed towards the three following interrelated objectives:

- **Ob_1a**: to provoke students’ production of personal signs related to the actual use of the artefact;
- **Ob_1b**: to construct a shared context for these signs (evocating of the actual context of use of the artefact);
- **Ob_1c**: to obtain from students a number of contributions as large as possible.

These objectives are complementary; the teacher’s actions are often meant to pursue all of them simultaneously, even if from time to time one of them might be prominent.

The following excerpt shows an example of the teacher’s intervention in a S1 situation

**Excerpt 1 1st Discussion**

12) Teacher: Well, then…let's see if looking back at what we have done, we can find what we want our idea of function to be…so…what have you done, tell me so I'll do it too [the teacher is ready to act on the computer]…who can tell me?

13) BA: I'll tell you…so…we drew points A, B, and P, anywhere and then we applied the macro construction effect to points A, B and P in this order and we got another point which we called H. [in the mean time, the teacher does the construction on the computer and the image is projected for the whole class]

At the beginning of the discussion, the teacher invites the students to come back to what they did and explicitly declares what has to be considered the common didactic goal of the activity: developing a shared idea of function (12). To do that, the teacher asks students to recall the task faced in the previous session and to report on how they accomplish it. In this particular case, a computer and a video-projector are available in the room and the teacher performs on it the construction described by BA: so, the realization of the task with the artefact accompanies BA’s recount. This does not always occur, and, the use of the artefact can be simply evoked. Though keeping the same objectives, the teacher can use the artefact even more indirectly: for instance explicitly referring to the written texts produced by the students after the lab activity

In any case we can recognize that the teacher “uses the artefact” for accomplishing his didactical tasks: promoting the production in a social context of signs related to the use of the artefact and constructing a common context for developing shared meanings for these signs.

As mentioned, the need of provoking the production of signs can present itself in different moments. The following excerpt, for instance, shows a later intervention of the teacher still meant to direct the discussion back to the request of the task.

**Excerpt 2 1st Discussion**
21) Teacher: Yes, because now you were lead to discover this construction...why? What was said? I mean, what were you asked to do?

22) BA: We had to say...first if we moved point A ... which were the points that moved and didn’t move...

These interventions are only partially planned in advance; they are mostly produced as on the spot reactions to students’ behaviour. The teacher recurs to this kind of interventions whenever he feels the need to recover the experience lived in the context of the artefact. Even if they may appear redundant, their recurrence serves to construct a rich net of meanings related to a specific sign (in this case, for instance, expressions involving “point” and “move”), and to enlarge the students’ participation so as to achieve a collaborative construction of meanings.

The above excerpts illustrate different instantiations of a same behaviour. Facing the same kind of situation (S1), the teachers identifies and selects the same kind of objective (Ob1 in its articulation) and “uses the artefact” in a consistent way: he asks the students to go back to the task and to make clear how they used the artefact for accomplishing it. Through this kind of request, the teacher fosters the reconstruction of the context of the task and of the modality of use of the artefact in that context.

The observation of these regularities in the organisation of the teacher’s activity leads us to infer the existence of a class of operational invariants piloting it. Among these invariants one can certainly list:

- **OpInv_1a**: Signs related to the actual use of an artefact can emerge after an explicit request of reporting on the activity with the artefact (referring to the classroom common experience with it).
- **OpInv_1b**: An explicit request of reporting on the activity with the artefact (referring to the classroom common experience) contributes to the re-construction of a common shared context.
- **OpInv_1c**: The common endeavour of communicating fosters the process of sharing signs

The “focalization” scheme

A productive utilization of the back to the task scheme may have the effect of provoking a large number of contributions and this may generate a lot of spurious elements. This originates a second class of situations that ask for the teacher to intervene.

- By S2 we define the class of situations when the discussion has led to the emergence and sharing of a rich net of signs related to the use of the artefact and there is the need of selecting the pertinent aspects of their shared meanings in respect to the development of the mathematical signs that constitute the final education goal.

Facing a situation S2, the teacher’s actions aim at achieving the following objectives:

- **Ob_2a**: to highlight specific (shared) signs produced up to that moment;
- **Ob_2b**: to select pertinent aspects of the meanings of these (shared) signs;
- **Ob_2c**: to circumscribe the reference of certain signs to specific aspects of the use of the artefact;
- **Ob_2d**: to support students’ consciousness-raising of these key aspects.

The next example shows the teacher facing a situation S2 (14) resulting exactly from a back to the task intervention (12, excerpt 1). In this case, the teacher draws the students’ attention towards the macro “effect 1” and asks them to provide an interpretation of that macro (14).

**Excerpt 3 1st Discussion**

14) Teacher: Ok, let’s stop here...there is something...I mean if I had to see this effect 1...what do you think the macro effect 1 is?

15) BA: I mean, it’s the construction that...there is a hidden construction behind it that allows us to...draw point H starting from points A, B, and P.

16) Teacher: Effect 1 condenses, hiding it, a construction that you then discovered...and what does this construction do?

17) BA: It constructs a point, it constructs point H...because we did...

18) Teacher: It constructs point H starting from?

19) Chorus: The three points.
Referring to the hidden construction (15), BA explains the characteristics of the functioning of a macro: it states a relation of dependency between the initial elements and the final ones. The teacher partially reformulates the explanation and pushes the student to express what the construction does, until the key elements become clear for everyone - “It constructs point H from the three points”.

The teacher’s intervention starts the process of (a) sharing the key aspects of the signs related to the artefact use and (b) raising the students’ awareness of these aspects; in addition he constantly feeds the process through echoing the students’ contributions (16 and 18).

The following excerpt (which incorporates the excerpt 2) contains the transcript of a dialog taking place immediately after the previous one. After a back to the task intervention (21) the teacher asks the students’ to recall how they experienced through the use of the dragging tool the dependence of movement of a point from the movement of another.

**Excerpt 4 1st Discussion**

21) Teacher: Yes, because now you were led to discover this construction…why? what was said? I mean, what “did you have to do”?

22) BA: We had to say…first if we moved point A which were the points that moved and didn’t move…

23) Teacher: Ok, then…for example, moving P, I see that only H moves and not only…I also see that what moves…?

The teacher intentionally promotes students’ awareness of those aspects of their experience that are significant for developing the mathematical meanings that are objective of his didactic project. In order to do that the teacher intervenes to make students focus on the key aspects of such experience. The main point is to limit the semantic field of specific signs and isolate those aspects that are pertinent in respect to the development of the desired mathematical signs. When facing a situation S2, in order to achieve the objectives Ob2 the teacher “uses the artefact” focalizing certain specific aspects of the students’ use of the artefact itself.

The above excerpts should give an idea of the possible actions that the teacher can perform to draw the students’ attention on particular elements of their experience. These actions do not involve only speech: gestures or changes in the tone of the voice are often observed showing the intentionality of focalizing.

Through the observation of these regularities the following operational invariants can be hypothesized:

- **OpInv_2a**: through specific forms of speech it is possible to direct students’ attention towards specific aspects of the use of the artefact;
- **OpInv_2b**: focusing on specific aspects of the use of the artefact helps circumscribing the meaning of specific signs;
- **OpInv_2c**: in a social context, the effort of making explicit the key aspects of the shared signs (through specific forms of speech) helps students to become aware of these elements and of the possibility of isolating them from the multiplicity of other aspects.

As emerges from the previous discussion, the two schemes (back to the task and focalization) are complementary: one is oriented to enrich the semantic field referring to the use of the artefact, whereas the other is oriented to select the pertinent aspects. We can observe a double movement from interventions of one type to interventions of the other.

### 5.7.2 Towards mathematical signs

The repeated alternate mobilization of the two schemes described above can lead to the joint construction of shared and stable signs, which on the one hand are anchored to the artefact actual, and on the other hand retain those key elements of their meanings which are pertinent in respect to the development of the desired mathematical signs.

The evolution towards the mathematical signs requires a re-elaboration of the produced signs and a de-contextualization from the artefact and its use. Two schemes can be mobilized for attaining this evolution.

**The “ask for a synthesis” scheme**

As an effect of the teacher’s activity, the original initial situation has been transformed into a new class of situations.
By S3 we define the class of situations when the discussion has led to the emergence of shared and stable signs condensing the key aspects of the common experience with the artefact, and there is the need of generalizing and de-contextualizing the meanings emerged.

The process of generalization and de-contextualization of meanings cannot consist of simply replacing the produced signs (e.g. "moving point") with the appropriate mathematical signs ("independent variable"). In fact through their evolution the signs should gain the full mathematical meaning keeping at the same time some of the crucial aspects coming from their origin. For instance, the dynamics intrinsic of a moving point is expected to remain as a component of the mathematical meaning of independent variable. This evolution cannot start but from the personal meanings which students attach to the shared signs. It is a complex semiotic process which asks for the teacher to intervene in order to:

- **Ob_3b**: promote the de-contextualization (from the use of the artefact) by the students of the meanings related to the use of the artefact,
- **Ob_3c**: promote the generalization (with respect to the specific tasks) by the students of the meanings related to the use of the artefact;
- **Ob_3d**: maintain (in the de-contextualization and generalization process) those aspects of the personal meanings, which are related to the use of the artefact but are recognised as pertinent to the target mathematical signs;

In order to trigger this, process the teacher encourages the students to synthesize, that is to report in a synthetic way what has been done and discussed in the classroom up to a certain moment.

**Excerpt 5 1st Discussion**

211) Teacher: *Who would like to synthesize all what I have said*? ... but I want someone that never talked ... MA!

212) MA: what I understood...?

213) Teacher: Ok, go on, what did you understand?

214) MA: I mean ... there are certain things that are taken from others that are independent... that are points A, B and P; H is obtained by a construction that derives from A, B and P, thus H depends on the position ...

215) Teacher: ... on the position of the three points A, B and P. *Thus the function... what is it [the function] for you?*

216) MA: The function for me is ... I mean it should be a construction that practically ... is obtained by different means ... that derive from ...

217) Teacher: *From which points?*

218) MA: A, B, and P.

219) Teacher: OK

The teacher explicitly asks for a synthesis (211). MA, who did not intervene before, starts clarifying his interpretation of the request: making explicit what he understood. The following utterance (214) shows traces of the expected process of de-contextualization. MA uses generic terms, although maintaining the link with the artefact context. The teacher follows the student when he comes back to the artefact context, but immediately restarts the mediation process and asks to make explicit the personal meaning of the sign <function> (215). MA attempts a general formulation without referring to the artefact. Because the link with the artefact context was not cut, it is possible to come back to it as soon as the formulation task becomes too difficult (217).

As the excerpt shows the evolution of signs may progress through an alternate movement: back and forth from the artefact context to the mathematical context. Expressions such as "certain things" or "depends on" seem to play a hinge-role, connecting the two contexts, but also fostering the move from one to the other.

The following operational invariants seem to underpin the teacher’s activity in this class of situations.

- **Oplnv_3b**: An explicit request of synthesizing may trigger a process of generalization and de-contextualization of meanings with respect to the actual use of the artefact;
- **Oplnv_3c**: syntheses can help forming a shared semiotic environment within which mathematical signs might be produced and put in relation with the artefact signs;
- **OplInv_3d**: a request of synthesis may foster the establishing of explicit connections between the artefact context and the mathematical context.

**The “provide a synthesis” scheme**

Generally speaking, the mobilization of the ask for a synthesis scheme is expected to contribute to the development of the shared semiotic environment within which mathematical signs might be produced and put in relation with the artefact signs. Within this context the teacher may introduce the point of view of mathematics, and eventually a standard terminology.

- By **S4** we define the class of situations, when the discussion has led to the de-contextualization and generalization of meanings form the context of use of the artefact towards the context of mathematics, and there is the need of ratifying the acceptability and the status of a sign within the mathematical context.

Crucial steps have been moved towards the formation of the desired mathematical signs, but the evolution cannot be considered complete yet. There is still the need of making explicit the mathematical status of certain signs, explicitly establishing connections between the artefact context and the related meanings, and the mathematical context and its meanings.

The objectives of the teacher in this situation are:

- **Ob_4a**: to provide a mathematical formulation which introduces the desired mathematical signs as evolution of the personal signs previously emerged;
- **Ob_4b**: to ratify the acceptability and the mathematical status of a specific sign;
- **Ob_4c**: to highlight the system of relationships between mathematical meanings, and meanings constructed trough the classroom discussion.

Below is an example showing the teacher who synthesizes what emerged up to that moment, and in doing so explains the relationship between independent and dependent variables. It is worth noticing that the experience with the artefact is still evoked by the teacher.

**Excerpt 6 1st Discussion**

159) Teacher : Well, then what happens is that in general for a function, the points from which I start are named independent variables, because I can move them wherever I like, whilst what I obtain is named dependent variable, because it depends … on what [does it depend]?

160) MO: [it depends on] the independent variables.

In this exemplar intervention the teacher fixes the use of mathematical terms such as “independent variable” and “dependent variable”. He makes explicit the link between the mathematical signs and the meanings that emerged from the activity with the artefact through the link with specific signs belonging to the artefact context: <point from which I start>, <points I can move wherever I like>. The way into the mathematics domain is opened.

It is clear that in order to pursue the objectives Ob4 the teacher is demanded of a more “direct” and evident involvement. In fact he has the responsibility to bring in the discourse the voice and the point of view of mathematics. An explicit intervention is needed to ratify the recognisability and acceptability of the developed signs for the mathematicians’ community.

With this respect, the teacher’s activity seems to reveal the action of different operational invariants:

- **OplInv_4a**: Once the process of generalization and de-contextualization has been triggered, it is possible to introduce mathematical signs, linking them to the shared meanings developed up to that moment;
- **OplInv_4b**: An explicit synthesis given by the teacher may provide a mathematical formulation of the meanings emerged;
- **OplInv_4c**: An orchestration of the classroom discussion performed through a continuous back and forth from the artefact context to the mathematical context may promote a rich texture of personal meanings and mathematical meanings.

The mobilization of the **Ask for a synthesis** and **Provide a synthesis** schemes have crucial importance for producing teacher’s interventions aimed at directly involving students in generalization and de-
contextualization processes, and, at the same time, giving them the possibility of appropriating of the mathematical signs that are introduced by the teacher.

5.8 Conclusions

Within the frame offered by the TSM the use of an artefact has a twofold nature: on the one hand it is directly used by the students as a means to accomplish a task; on the other hand it is indirectly used by the teacher as a means to achieve specific educational goals. Using the terminology from (Winslow, 2003, p.275), we can say that a specific artefact may play both a pragmatic and a didactical role. In this sense, a specific artefact is to be considered a resource for the teacher, that is “a means to support” his didactical action. In this chapter, we developed a description of how the teacher can pilot a semiotic process based on the use of an artefact. More specifically, taking an instrumental approach, we outlined the utilization schemes that may be put in place to promote the evolution of signs during a collective discussion. According to the analysis of our data, these schemes showed their effectiveness in terms of the TSM, that is they might assure the functioning of an artefact as an instrument of semiotic mediation.

As appears, the schemes described are not specific of the particular artefact or of the mathematical domain considered. This is due to the level of description retained, which in turn is motivated by the intention of carrying out a discussion having a scope as wide as possible. It seems possible that retaining different levels of analysis one can highlight schemes directly related to the use of Cabri, and to the notions at stake.

Beyond illustrating the functioning of the semiotic mediation process, the previous elaboration intends to provide a contribution to a key issue concerning the notion of mediation, and specifically of semiotic mediation. In fact, as Kozulin (2003) pointed out, it is important to identify different forms of mediation to be related to teaching-learning process. Particularly it seems important to identify and express them in an explicit way so that they can be communicated and shared in the community of the teachers. In this respect, the combination of an artefact and of the specific semiotic mediation schemes of utilization could be considered the product of an instrumental genesis related to a resource for the teachers.

The study of the process of development of an instrument of semiotic mediation shares many characteristics with the study of process of the documentary genesis (Gueudet and Trouche, chapter 3). But there are also some important differences, at least at the current level of elaboration. On the one hand in our analysis we only focused on the teacher’s use of a specific artefact and left out the study of the use of the other resources (students’ written productions or other artefacts). The study of the documentational genesis would have demanded to take into consideration as far as possible the whole system of resources. On the other hand our analysis did not mean to classify or describe the teacher’s utilization schemes of an artefact tout-court: in the case of an instrument of semiotic mediation, the schemes of utilization emerge and make sense within the specific theoretical frame of semiotic mediation.

What makes this analysis valuable for us is the fact that it provides an explicit model of the teacher’s actions mediated by the artefact, thus shading light on specific elements concerning the orchestration of the classroom discussion, and clarifying what is expected by the teacher because the artefact may function as an instrument of semiotic mediation. Although very specific and tightly framed by the TSM, the model presented in this chapter contributes to elaborate on the didactic process with a focus on the teacher. Regarding this, a meaningful issue concerns the relevance of teacher’s consciousness about his own role and specifically about the choices that he has to undertake. This brings forth the need of clarifying teachers’ direct involvement in the integration of ICT. The analysis and the description of teachers’ actions is the base on which math educators may contribute to foster teachers’ professional development along this dimension. Our study is meant to contribute in this respect, on the one hand providing a clear frame within which the use of an artefact as a didactic resource can be described, and on the other hand providing a description of fruitful schemes of utilization that can be employed to implement that resource in school practice.

References


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Gueudet, G., & Trouche, L. CHAPTER 3 THIS VOLUME


Ruthven CHAPTER 10 THIS VOLUME

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